M3 January 2007

1. A particle P moves along the x-axis. At time $t = 0$, P passes through the origin O, moving in the positive x-direction. At time t seconds, the velocity of P is v m s^{-1} and $OP = x$ metres. The acceleration of P is $\frac{1}{12}(30 - x)$ m s^{-2}, measured in the positive x-direction.

(a) Give a reason why the maximum speed of P occurs when $x = 30$.

(1)

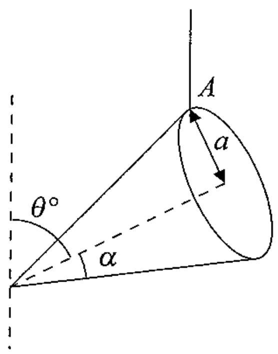
Given that the maximum speed of P is 10 m s^{-1},

(b) find an expression for v^2 in terms of x.

(5)

2.

Figure 1



A uniform solid right circular cone has base radius a and semi-vertical angle α, where $\tan\alpha = \frac{1}{3}$. The cone is freely suspended by a string attached at a point A on the rim of its base, and hangs in equilibrium with its axis of symmetry making an angle of $\theta°$ with the upward vertical, as shown in Figure 1.

Find, to one decimal place, the value of θ.

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3. A particle P of mass m is attached to one end of a light elastic string, of natural length a and modulus of elasticity $3.6mg$. The other end of the string is fixed at a point O on a rough horizontal table. The particle is projected along the surface of the table from O with speed $\sqrt{(2ag)}$. At its furthest point from O, the particle is at the point A, where $OA = \frac{4}{3}a$.

(a) Find, in terms of m, g and a, the elastic energy stored in the string when P is at A.

(3)

(b) Using the work-energy principle, or otherwise, find the coefficient of friction between P and the table.

(6)

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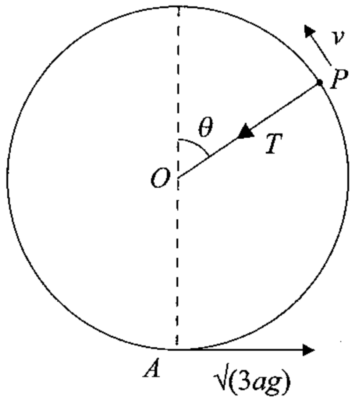
4. **Figure 2**



A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a point O. The point A is vertically below O, and $OA = a$. The particle is projected horizontally from A with speed $\sqrt{(3ag)}$. When OP makes an angle θ with the upward vertical through O and the string is still taut, the tension in the string is T and the speed of P is v, as shown in Figure 2.

(a) Find, in terms of a, g and θ, an expression for v^2.

(3)

(b) Show that $T = (1 - 3\cos\theta)mg$.

(3)

The string becomes slack when P is at the point B.

(c) Find, in terms of a, the vertical height of B above A.

(2)

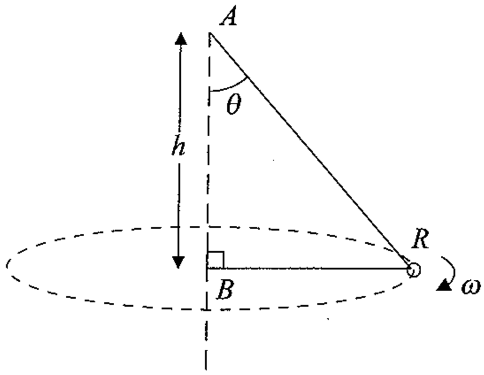
After the string becomes slack, the highest point reached by P is C.

(d) Find, in terms of a, the vertical height of C above B.

(5)

5.

Figure 3



One end of a light inextensible string is attached to a fixed point A. The other end of the string is attached to a fixed point B, vertically below A, where $AB = h$. A small smooth ring R of mass m is threaded on the string. The ring R moves in a horizontal circle with centre B, as shown in Figure 3. The upper section of the string makes a constant angle θ with the downward vertical and R moves with constant angular speed ω. The ring is modelled as a particle.

(a) Show that $\omega^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)$.

(7)

(b) Deduce that $\omega > \sqrt{\frac{2g}{h}}$.

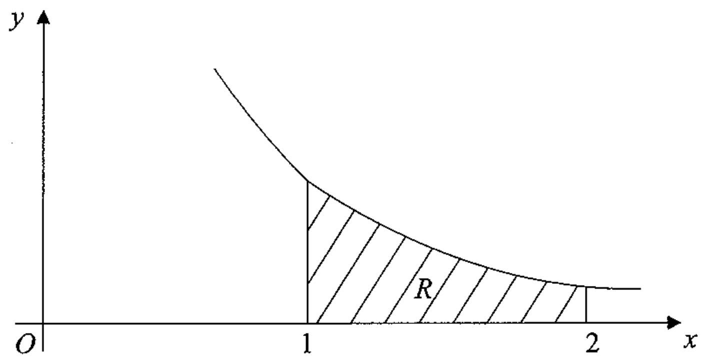
(2)

Given that $\omega = \sqrt{\frac{3g}{h}}$,

(c) find, in terms of m and g, the tension in the string.

(4)

6. **Figure 4**

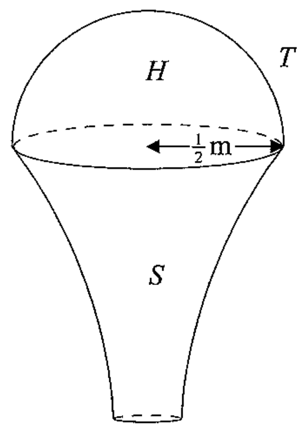


The shaded region R is bounded by the curve with equation $y = \frac{1}{2x^2}$, the x-axis and the lines $x = 1$ and $x = 2$, as shown in Figure 4. The unit of length on each axis is 1 m. A uniform solid S has the shape made by rotating R through 360° about the x-axis.

(a) Show that the centre of mass of S is $\frac{2}{7}$ m from its larger plane face.

(6)

Figure 5



A sporting trophy T is a uniform solid hemisphere H joined to the solid S. The hemisphere has radius $\frac{1}{2}$ m and its plane face coincides with the larger plane face of S, as shown in Figure 5. Both H and S are made of the same material.

(b) Find the distance of the centre of mass of T from its plane face.

(7)

7. A particle P of mass 0.25 kg is attached to one end of a light elastic string. The string has natural length 0.8 m and modulus of elasticity λ N. The other end of the string is attached to a fixed point A. In its equilibrium position, P is 0.85 m vertically below A.

(a) Show that $\lambda = 39.2$.

(2)

The particle is now displaced to a point B, 0.95 m vertically below A, and released from rest.

(b) Prove that, while the string remains stretched, P moves with simple harmonic motion of period $\frac{\pi}{7}$ s.

(6)

(c) Calculate the speed of P at the instant when the string first becomes slack.

(3)

The particle first comes to instantaneous rest at the point C.

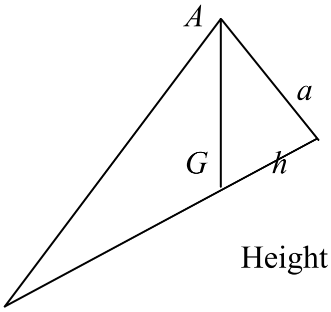
(d) Find, to 3 significant figures, the time taken for P to move from B to C.

(5)

TOTAL FOR PAPER: 75 MARKS

January 2007
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
1.	(a) Maximum speed when accel. = 0 (o.e.)	B1 (1)
	(b) $\frac{1}{12}(30-x) = v\frac{\mathrm{d}v}{\mathrm{d}x}$ (acceln = … + attempt to integrate)	M1 ↓
	Use of $v\frac{\mathrm{d}v}{\mathrm{d}x}$: $\frac{v^2}{2} = \frac{1}{12}\left(30x - \frac{x^2}{2}\right)$ $(+c)$	M1 A1 ↓
	Substituting $x = 30$, $v = 10$ and finding c (= 12.5), or limits	M1
	$v^2 = 25 + 5x - \frac{1}{12}x^2$ (o.e.)	A1 (5)
	(a) Allow "acceln > 0 for $x < 30$, acceln < 0 for $x > 30$" Also "accelerating for $x < 30$, decelerating for $x > 30$" But "acceln < 0 for $x > 30$" only is B0	
	(b) 1st M1 will be generous for wrong form of acceln (e.g. dv/dx)! 3rd M1 If use limits, they must use them in correct way with correct values Final A1. Have to accept any expression, but it must be for v^2 explicitly (not $1/2v^2$), and if in separate terms, one can expect like terms to be collected. Hence answer in form as above, or e.g. $\frac{1}{12}(300 + 60x - x^2)$; also $100 - \frac{1}{12}(30-x)^2$	

2.	 Height of cone = $\dfrac{a}{\tan\alpha} = 3a$ Hence $h = \frac{3}{4}a$ $\tan\theta = \dfrac{a}{\frac{3}{4}a} = \dfrac{4}{3} \Rightarrow \theta = 53.1°$ 1st M1 (generous) allow any trig ratio to get height of cone (e.g. using sin) 3rd M1 For correct trig ratio on a suitable triangle to get θ or complement (even if they call the angle by another name – hence if they are aware or not that they are getting the required angle)	M1 A1 ↓ M1 ↓ M1 A1 (5)

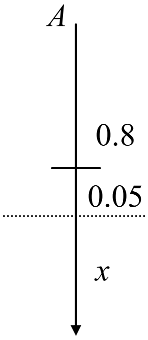
3	(a) E.P.E. $= \frac{1}{2}\frac{3.6mg}{a}x^2 = \frac{1}{2}\frac{3.6mg}{a}\left(\frac{a}{3}\right)^2$	M1 A1
	$= \underline{0.2\ mga}$	A1 (3)
	(b) Friction = $\mu mg \Rightarrow$ work done by friction = $\mu mg\left(\frac{4a}{3}\right)$	M1 A1
	Work-energy: $\frac{1}{2}m.2ga = \mu mgd + 0.2\ mga$ (3 relevant terms)	M1 A1√ ↓
	Solving to find μ: $\underline{\mu = 0.6}$	M1 A1 (6)
	(b) 1st M1: allow for attempt to find work done by frictional force (i.e. not just finding friction). 2nd M1: "relevant" terms, i.e. energy or work terms! A1 f.t. on their work done by friction	

4.	(a) Energy: $\frac{1}{2}m.3ag - \frac{1}{2}mv^2 = mga(1+\cos\theta)$	M1 A1
	$\underline{v^2 = ag(1-2\cos\theta)}$ **(o.e.)**	A1 (3)
	(b) $T + mg\cos\theta = m\frac{v^2}{a}$	M1 A1
	Hence $\underline{T = (1-3\cos\theta)mg}$ (*)	A1 cso (3)
	(c) Using $T = 0$ to find $\cos\theta$	M1
	Hence height above A = $\underline{\frac{4}{3}a}$ Accept 1.33a (but must have 3+ s.f.)	A1 (2)
	(d) $v^2 = \frac{1}{3}ag$ **(o.e.)** f.t. using $\cos\theta = \frac{1}{3}$ in v^2	B1√
	consider vert motion: $(v\sin\theta)^2 = 2gh$ (with v resolved)	M1 A1 ↓ M1
	$\sin^2\theta = \frac{8}{9}$ (or θ = 70.53, sin θ = 0.943) and solve for h (as ka)	
	$h = \underline{\frac{4}{27}a}$ or 0.148a (awrt)	A1
	OR consider energy: $\frac{1}{2}m(v\cos\theta)^2 + mgh = \frac{1}{2}mv^2$ (3 non-zero terms)	M1 A1 ↓ M1
	Sub for v, θ and solve for h	
	$h = \underline{\frac{4}{27}a}$ or 0.148a (awrt)	A1

Question Number	Scheme	Marks
5.	(a) $\updownarrow\ T\cos\theta = mg$	B1
	$\leftrightarrow\ T + T\sin\theta = mr\omega^2$ (3 terms)	M1 A1
	$r = h\tan\theta$	B1
	$\frac{mg}{\cos\theta}(1+\sin\theta) = \frac{m\omega^2 h\sin\theta}{\cos\theta}$ (eliminate r)	↓ M1
	$\underline{\omega^2 = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right)}$ (*) (solve for ω^2)	↓ M1 A1 (7)
	(b) $\omega^2 = \frac{g}{h}\left(\frac{1}{\sin\theta}+1\right) > \frac{2g}{h}$ ($\sin\theta < 1$) $\Rightarrow \omega > \sqrt{\frac{2g}{h}}$ (*)	M1 A1 (2)
	(c) $\frac{3g}{h} = \frac{g}{h}\left(\frac{1+\sin\theta}{\sin\theta}\right) \Rightarrow \sin\theta = \frac{1}{2}$	M1 A1
	$T\cos\theta = mg \Rightarrow \underline{T = \frac{2\sqrt{3}}{3}mg}$ or $\underline{1.15mg}$ (awrt)	↓ M1 A1 (4)
	(a) Allow first B1 M1 A1 if assume different tensions (so next M1 is effectively for eliminating *r* **and** *T*. (b) M1 requires a *valid* attempt to derive an *in*equality for ω. (Hence putting sin θ = 1 immediately into expression of ω^2 [assuming this is the critical value] is M0.)	

6.	(a) Moments: $\pi \int_1^2 xy^2 \mathrm{d}x = V\bar{x}$ or $\int_1^2 xy^2\mathrm{d}x = \bar{x}\int_1^2 y^2\mathrm{d}x$	M1
	$\int_1^2 y^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^4}\mathrm{d}x = \left[-\frac{1}{12x^3}\right]_1^2 \quad (= \frac{7}{96})$ (either)	M1 A1
	$\int_1^2 xy^2\,\mathrm{d}x = \int_1^2 \frac{1}{4x^3}\mathrm{d}x = \left[-\frac{1}{8x^2}\right]_1^2 \quad (= \frac{3}{32})$ (both)	A1
		↓
	Solving to find $\bar{x}$ $(= \frac{9}{7})$ $\Rightarrow$ required dist $= \frac{9}{7} - 1 = \frac{2}{7}$ m (*)	M1 A1 cso (6)
	(b)	
	H, S, T	
	Mass: H $(\rho)\,\frac{2}{3}\pi\left(\frac{1}{2}\right)^3$ $\left[= \frac{1}{12}(\rho)\pi\right]$, S $(\rho)\,\frac{7\pi}{96}$, T $H + S$ $\left[= \frac{5}{32}(\rho)\pi\right]$	B1, M1
	Dist of CM from base: H $\frac{19}{16}$ m, S $\frac{5}{7}$ m, T $\bar{x}$	B1 B1
	Moments: $\left[= \frac{1}{12}(\rho)\pi\right]\left(\frac{19}{16}\right) + (\rho)\,\frac{7\pi}{96}\left(\frac{5}{7}\right) = \left[\frac{5}{32}(\rho)\pi\right]\bar{x}$	M1 A1
	$\bar{x} = \frac{29}{30}$ m or 0.967 m (awrt)	A1 (7)
	Allow distances to be found from different base line if necessary	

7. (a)



$T = \frac{\lambda}{0.8}(0.05) = 0.25g$ — M1

$\lambda = \frac{(0.8)(0.25g)}{0.05} = 39.2$ (*) — A1 (2)

(b) $T = \frac{39.2}{0.8}(x + 0.05)$ — M1

$mg - T = ma$ (3 term equn) — M1

$0.25g - \frac{39.2}{0.8}(x + 0.05) = 0.25\,\ddot{x}$ (or equivalent) — A1

$\ddot{x} = -196\,x$ — A1

SHM with period $\frac{2\pi}{\omega} = \frac{2\pi}{14} = \frac{\pi}{7}$ s (*) — ↓ M1 A1 cso (6)

(c) $v = 14\sqrt{\{(0.1)^2 - (0.05)^2\}}$ — M1 A1√

$= 1.21(24\ldots) \approx$ <u>1.21 m s^{-1}</u> (3 s.f.) Accept $7\sqrt{3}/10$ — A1 (3)

(d) Time T under gravity $= \frac{1.21..}{g}$ ($= 0.1237\,s$) — B1√

Complete method for time T' from B to slack.

[↑ e.g. $\frac{\pi}{28} + t$, where $0.05 = 0.1\sin 14t$

OR T', where $-0.05 = 0.1\cos 14T'$] — M1 A1

$T'' = 0.1496$s — A1

Total time = $T + T'$ = <u>0.273 s</u> — A1 (5)

(b) 1st M1 must have extn as $x + k$ with $k \neq 0$ (but allow M1 if e.g. $x + 0.15$), or must justify later

For last four marks, *must* be using $\ddot{x}$ (not a)

(c) Using $x = 0$ is M0

(d) M1 – must be using distance for when string goes slack. Using $x = -0.1$ (i.e. assumed end of the oscillation) is M0